2001 AMC 10

Time limit: 75 minutes

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https://live.poshenloh.com/past-contests/amc10/2001

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1. The median of the list

$$n,\ n+3,\ n+4,\ n+5,\ n+6,$$
$$n+8,\ n+10,\ n+12,\ n+15$$

is 10. What is the mean?

A 4

B 6

C 7

D 10

E 11

2. A number x is 2 more than the product of its reciprocal and its additive inverse. In which interval does the number lie?

A $-4 \leq x \leq -2$

B $-2 < x \leq 0$

C $0 < x \leq 2$

D $2 < x \leq 4$

E $4 < x \leq 6$

3. The sum of two numbers is S. Suppose 3 is added to each number and then each of the resulting numbers is doubled. What is the sum of the final two numbers?

A $2S + 3$

B $3S + 2$

C $3S + 6$

D $2S + 6$

E $2S + 12$

4. What is the maximum number of possible points of intersection of a circle and a triangle?

A 2

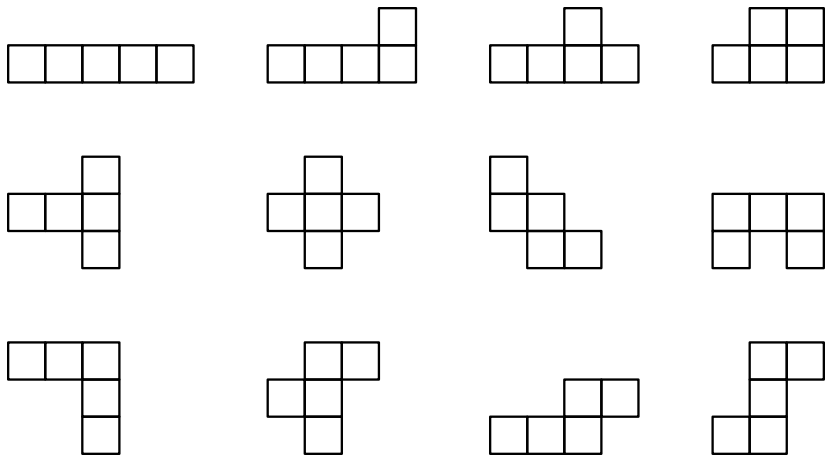
B 3

C 4

D 5

E 6

5. How many of the twelve pentominoes pictured below have at least one line of symmetry?



A 3

B 4

C 5

D 6

E 7

6. Let $P(n)$ and $S(n)$ denote the product and the sum, respectively, of the digits of the integer n. For example, $P(23) = 6$ and $S(23) = 5$. Suppose N is a two-digit number such that $N = P(N) + S(N)$. What is the units digit of N?

A 2

B 3

C 6

D 8

E 9

7. When the decimal point of a certain positive decimal number is moved four places to the right, the new number is four times the reciprocal of the original number. What is the original number?

A 0.0002

B 0.002

C 0.02

D 0.2

E 2

8. Wanda, Darren, Beatrice, and Chi are tutors in the school math lab. Their schedule is as follows: Darren works every third school day, Wanda works every fourth school day, Beatrice works every sixth school day, and Chi works every seventh school day. Today they are all working in the math lab. In how many school days from today will they next be together tutoring in the lab?

A 42

B 84

C 126

D 178

E 252

9. The state income tax where Kristin lives is levied at the rate of $p\%$ of the first $\$28000$ of annual income plus $(p+2)\%$ of any amount above $\$28000$. Kristin noticed that the state income tax she paid amounted to $(p+0.25)\%$ of her annual income. What was her annual income?

A $\$28000$

B $\$32000$

C $\$35000$

D $\$42000$

E $\$56000$

10. If x, y, and z are positive with $xy = 24$, $xz = 48$, and $yz = 72$, then $x + y + z$ is

A 18

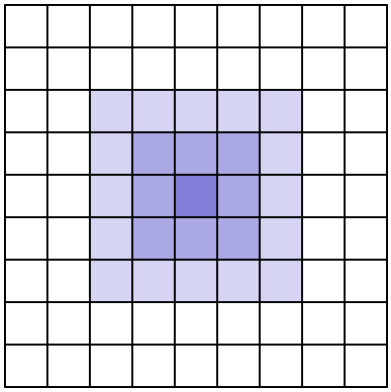
B 19

C 20

D 22

E 24

11. Consider the dark square in an array of unit squares, part of which is shown. The first ring of squares around this center square contains 8 unit squares. The second ring contains 16 unit squares. If we continue this process, the number of unit squares in the 100th ring is



A 396

B 404

C 800

D $10{,}000$

E $10{,}404$

12. Suppose that n is the product of three consecutive integers and that n is divisible by 7. Which of the following is not necessarily a divisor of n?

A 6

B 14

C 21

D 28

E 42

13. A telephone number has the form $ABC-DEF-GHIJ$, where each letter represents a different digit. The digits in each part of the number are in decreasing order; that is, $A > B > C$, $D > E > F$, and $G > H > I > J$. Furthermore, D, E, and F are consecutive even digits; G, H, I, and J are consecutive odd digits; and $A + B + C = 9$. Find A.

A 4

B 5

C 6

D 7

E 8

14. A charity sells 140 benefit tickets for a total of $\$2001$. Some tickets sell for full price (a whole dollar amount), and the rest sell for half price. How much money is raised by the full-price tickets?

A $\$782$

B $\$986$

C $\$1158$

D $\$1219$

E $\$1449$

15. A street has parallel curbs 40 feet apart. A crosswalk bounded by two parallel stripes crosses the street at an angle. The length of the curb between the stripes is 15 feet and each stripe is 50 feet long. Find the distance, in feet, between the stripes.

A 9

B 10

C 12

D 15

E 25

16. The mean of three numbers is 10 more than the least of the numbers and 15 less than the greatest. The median of the three numbers is 5. What is their sum?

A 5

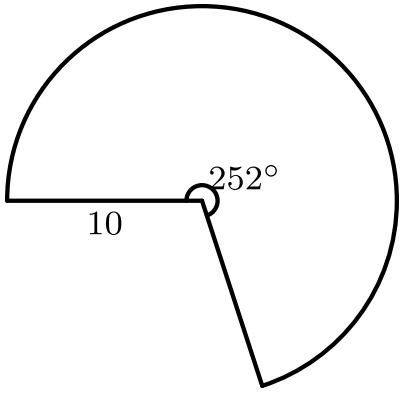
B 20

C 25

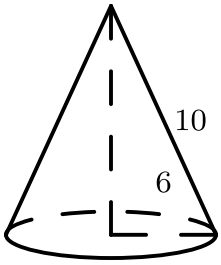
D 30

E 36

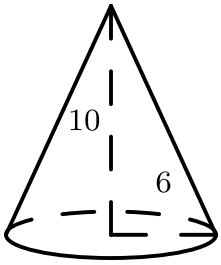
17. Which of the cones below can be formed from a $252°$ sector of a circle of radius 10 by aligning the two straight sides?



A



B



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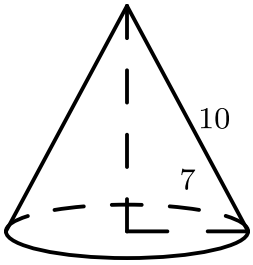

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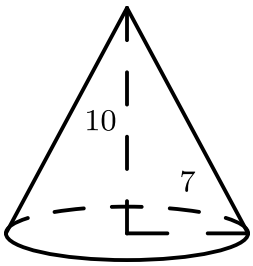
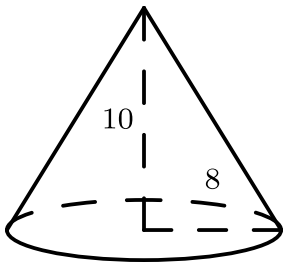
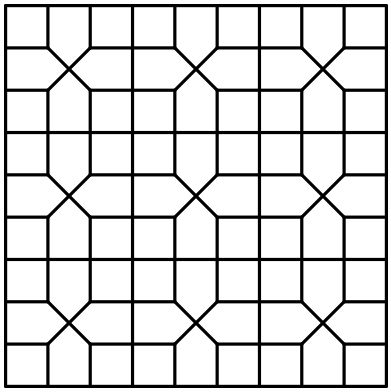


E



18. The plane is tiled by congruent squares and congruent pentagons as indicated. The percent of the plane that is enclosed by the pentagons is closest to



A 50

B 52

C 54

D 56

E 58

19. Pat wants to buy four donuts from an ample supply of three types of donuts: glazed, chocolate, and powdered. How many different selections are possible?

A 6

B 9

C 12

D 15

E 18

20. A regular octagon is formed by cutting an isosceles right triangle from each of the corners of a square with sides of length 2000. What is the length of each side of the octagon?

A $\frac{1}{3}(2000)$

B $2000(\sqrt{2}-1)$

C $2000(2-\sqrt{2})$

D 1000

E $1000\sqrt{2}$

21. A right circular cylinder with its diameter equal to its height is inscribed in a right circular cone. The cone has diameter 10 and altitude 12, and the axes of the cylinder and cone coincide. Find the radius of the cylinder.

A $\frac{8}{3}$

B $\frac{30}{11}$

C 3

D $\frac{25}{8}$

E $\frac{7}{2}$

22. In the magic square shown, the sums of the numbers in each row, column, and diagonal are the same. Five of these numbers are represented by v, w, x, y, and z. Find $y + z$.

v	24	w
18	x	y
25	z	21

A 43

B 44

C 45

D 46

E 47

23. A box contains exactly five chips, three red and two white. Chips are randomly removed one at a time without replacement until all the red chips are drawn or all the white chips are drawn. What is the probability that the last chip drawn is white?

A $\frac{3}{10}$

B $\frac{2}{5}$

C $\frac{1}{2}$

D $\frac{3}{5}$

E $\frac{7}{10}$

24. In trapezoid $ABCD$, $\overline{AB}$ and $\overline{CD}$ are perpendicular to $\overline{AD}$, with $AB + CD = BC$, $AB < CD$, and $AD = 7$. What is $AB \cdot CD$?

A 12

B 12.25

C 12.5

D 12.75

E 13

25. How many positive integers not exceeding 2001 are multiples of 3 or 4 but not 5?

A 768

B 801

C 934

D 1067

E 1167

Solutions: https://live.poshenloh.com/past-contests/amc10/2001/solutions