2002 AMC 10A

Time limit: 75 minutes

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https://live.poshenloh.com/past-contests/amc10/2002A

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1. The ratio

$$\frac{10^{2000}+10^{2002}}{10^{2001}+10^{2001}}$$

is closest to which of the following numbers?

A 0.1

B 0.2

C 1

D 5

E 10

2. For the nonzero numbers a, b, and c, define

$$(a, b, c) = \frac{a}{b} + \frac{b}{c} + \frac{c}{a}.$$

Find $(2, 12, 9)$.

A 4

B 5

C 6

D 7

E 8

3. According to the standard convention for exponentiation,

$$2^{2^{2^2}} = 2^{\left(2^{\left(2^2\right)}\right)} = 2^{16} = 65{,}536.$$

If the order in which the exponentiations are performed is changed, how many *other* values are possible?

A 0

B 1

C 2

D 3

E 4

4. For how many positive integers m does there exist at least one positive integer n such that $m \cdot n \leq m + n$?

A 4

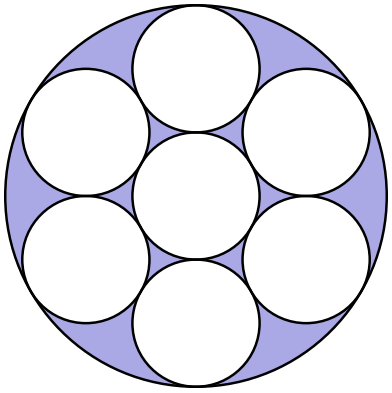
B 6

C 9

D 12

E infinitely many

5. Each of the small circles in the figure has radius one. The innermost circle is tangent to the six circles that surround it, and each of those circles is tangent to the large circle and to its small-circle neighbors. Find the area of the shaded region.



A π

B 1.5π

C 2π

D 3π

E 3.5π

6. Cindy was asked by her teacher to subtract 3 from a certain number and then divide the result by 9. Instead, she subtracted 9 and then divided the result by 3, giving an answer of 43. What would her answer have been had she worked the problem correctly?

A 15

B 34

C 43

D 51

E 138

7. If an arc of 45° on circle A has the same length as an arc of 30° on circle B, then the ratio of the area of circle A to the area of circle B is

A $\frac{4}{9}$

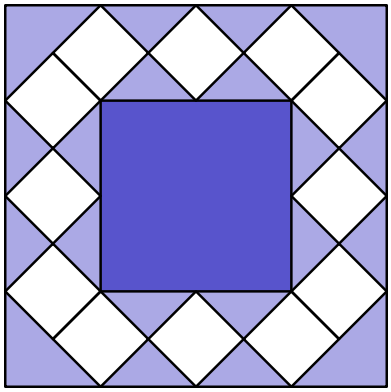
B $\frac{2}{3}$

C $\frac{5}{6}$

D $\frac{3}{2}$

E $\frac{9}{4}$

8. Betsy designed a flag using blue triangles, small white squares, and a red center square, as shown. Let B be the total area of the blue triangles, W the total area of the white squares, and R the area of the red square. Which of the following is correct?



A $B = W$

B $W = R$

C $B = R$

D $3B = 2R$

E $2R = W$

9. Suppose $A, B,$ and C are three numbers for which $1001C - 2002A = 4004$ and $1001B + 3003A = 5005$. The average of the three numbers $A, B,$ and C is

A 1

B 3

C 6

D 9

E not uniquely determined

10. Compute the sum of all the roots of $(2x+3)(x-4)+(2x+3)(x-6)=0$.

A $\frac{7}{2}$

B 4

C 5

D 7

E 13

11. Jamal wants to store 30 computer files on floppy disks, each of which has a capacity of 1.44 megabytes (mb). Three of his files require 0.8 mb of memory each, 12 more require 0.7 mb each, and the remaining 15 require 0.4 mb each. No file can be split between floppy disks. What is the minimal number of floppy disks that will hold all the files?

A 12

B 13

C 14

D 15

E 16

12. Mr. Earl E. Bird leaves his house for work at exactly $8:00$ A.M. every morning. When he averages 40 miles per hour, he arrives at his workplace three minutes late. When he averages 60 miles per hour, he arrives three minutes early. At what average speed, in miles per hour, should Mr. Bird drive to arrive at his workplace precisely on time?

A 45

B 48

C 50

D 55

E 58

13. The sides of a triangle have lengths of $15, 20,$ and $25.$ Find the length of the shortest altitude.

A 6

B 12

C 12.5

D 13

E 15

14. Both roots of the quadratic equation $x^2 - 63x + k = 0$ are prime numbers. The number of possible values of k is

A 0

B 1

C 2

D 4

E more than four

15. The digits $1, 2, 3, 4, 5, 6, 7,$ and 9 are used to form four two-digit prime numbers, with each digit used exactly once. What is the sum of these four primes?

A 150

B 160

C 170

D 180

E 190

16. If $a+1=b+2=c+3=d+4=a+b+c+d+5$, then $a+b+c+d$ is

A -5

B $-\frac{10}{3}$

C $-\frac{7}{3}$

D $\frac{5}{3}$

E 5

17. Sarah pours four ounces of coffee into an eight-ounce cup and four ounces of cream into a second cup of the same size. She then transfers half the coffee from the first cup to the second and, after stirring thoroughly, transfers half the liquid in the second cup back to the first. What fraction of the liquid in the first cup is now cream?

A $\frac{1}{4}$

B $\frac{1}{3}$

C $\frac{3}{8}$

D $\frac{2}{5}$

E $\frac{1}{2}$

18. A $3 \times 3 \times 3$ cube is formed by gluing together 27 standard cubical dice. (On a standard die, the sum of the numbers on any pair of opposite faces is 7.) The smallest possible sum of all the numbers showing on the surface of the $3 \times 3 \times 3$ cube is

A 60

B 72

C 84

D 90

E 96

19. Spot's doghouse has a regular hexagonal base that measures one yard on each side. He is tethered to a vertex with a two-yard rope. What is the area, in square yards, of the region outside the doghouse that Spot can reach?

A $\frac{2}{3}\pi$

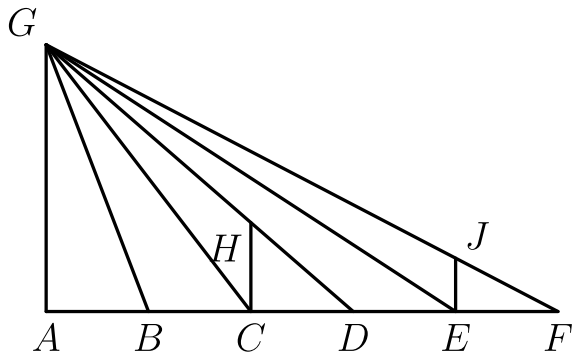
B 2π

C $\frac{5}{2}\pi$

D $\frac{8}{3}\pi$

E 3π

20. Points A, B, C, D, E, and F lie, in that order, on $\overline{AF}$, dividing it into five segments, each of length 1. Point G is not on line AF. Point H lies on $\overline{GD}$, and point J lies on $\overline{GF}$. The line segments $\overline{HC}$, $\overline{JE}$, and $\overline{AG}$ are parallel. Find $\frac{HC}{JE}$.



A $\frac{5}{4}$

B $\frac{4}{3}$

C $\frac{3}{2}$

D $\frac{5}{3}$

E 2

21. The mean, median, unique mode, and range of a collection of eight integers are all equal to 8. The largest integer that can be an element of this collection is

A 11

B 12

C 13

D 14

E 15

22. A set of tiles numbered 1 through 100 is modified repeatedly by the following operation: remove all tiles numbered with a perfect square, and renumber the remaining tiles consecutively starting with 1. How many times must the operation be performed to reduce the number of tiles in the set to one?

A 10

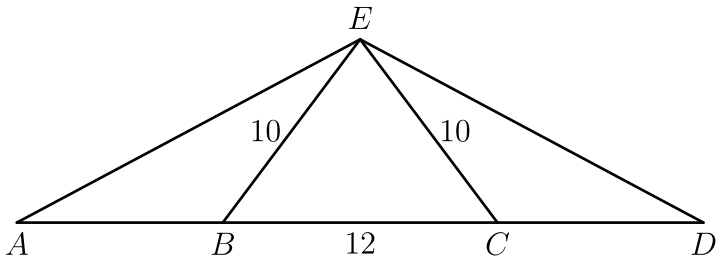
B 11

C 18

D 19

E 20

23. Points $A, B, C,$ and D lie on a line, in that order, with $AB = CD$ and $BC = 12$. Point E is not on the line, and $BE = CE = 10$. The perimeter of $\triangle AED$ is twice the perimeter of $\triangle BEC$. Find AB.



A $\frac{15}{2}$

B 8

C $\frac{17}{2}$

D 9

E $\frac{19}{2}$

24. Tina randomly selects two distinct numbers from the set $\{1, 2, 3, 4, 5\}$, and Sergio randomly selects a number from the set $\{1, 2, \ldots, 10\}$. The probability that Sergio's number is larger than the sum of the two numbers chosen by Tina is

A $\frac{2}{5}$

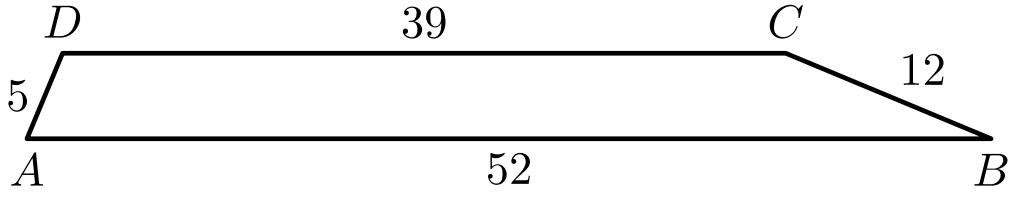
B $\frac{9}{20}$

C $\frac{1}{2}$

D $\frac{11}{20}$

E $\frac{24}{25}$

25. In trapezoid $ABCD$ with bases $\overline{AB}$ and $\overline{CD}$, we have $AB = 52, BC = 12, CD = 39$, and $DA = 5$. The area of $ABCD$ is



A 182

B 195

C 210

D 234

E 260

Solutions: https://live.poshenloh.com/past-contests/amc10/2002A/solutions

