2008 AMC 10B

Time limit: 75 minutes

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1. A basketball player made 5 baskets during a game. Each basket was worth either 2 or 3 points. How many different numbers could represent the total points scored by the player?

   A 2

   B 3

   C 4

   D 5

   E 6

2. A 4×4 block of calendar dates is shown. The order of the numbers in the second row is to be reversed. Then the order of the numbers in the fourth row is to be reversed. Finally, the numbers on each diagonal are to be added. What will be the positive difference between the two diagonal sums?

1	2	3	4
8	9	10	11
15	16	17	18
22	23	24	25

A 2

B 4

C 6

D 8

E 10

3. Assume that x is a positive real number. Which is equivalent to

$$\sqrt[3]{x\sqrt{x}}\,?$$

A $x^{\frac{1}{6}}$

B $x^{\frac{1}{4}}$

C $x^{\frac{3}{8}}$

D $x^{\frac{1}{2}}$

E x

4. A semipro baseball league has teams with 21 players each. League rules state that a player must be paid at least $\$15{,}000$, and that the total of all players' salaries for each team cannot exceed $\$700{,}000$. What is the maximum possible salary, in dollars, for a single player?

A $270{,}000$

B $385{,}000$

C $400{,}000$

D $430{,}000$

E $700{,}000$

5. For real numbers a and b, define $a \$ b = (a-b)^2$. What is $(x-y)^2 \$ (y-x)^2$?

A 0

B x^2+y^2

C $2x^2$

D $2y^2$

E $4xy$

6. Points B and C lie on $\overline{AD}$. The length of $\overline{AB}$ is 4 times the length of $\overline{BD}$, and the length of $\overline{AC}$ is 9 times the length of $\overline{CD}$. The length of $\overline{BC}$ is what fraction of the length of $\overline{AD}$?

A $\frac{1}{36}$

B $\frac{1}{13}$

C $\frac{1}{10}$

D $\frac{5}{36}$

E $\frac{1}{5}$

7. An equilateral triangle of side length 10 is completely filled in by non-overlapping equilateral triangles of side length 1. How many small triangles are required?

A 10

B 25

C 100

D 250

E 1000

8. A class collects $\$50$ to buy flowers for a classmate who is in the hospital. Roses cost $\$3$ each, and carnations cost $\$2$ each. No other flowers are to be used. How many different bouquets could be purchased for exactly $\$50$?

A 1

B 7

C 9

D 16

E 17

9. A quadratic equation $ax^2 - 2ax + b = 0$ has two real solutions. What is the average of the solutions?

A 1

B 2

C $\frac{b}{a}$

D $\frac{2b}{a}$

E $\sqrt{2a - b}$

10. Points A and B are on a circle of radius 5 and $AB = 6$. Point C is the midpoint of the minor arc AB. What is the length of the line segment AC?

A $\sqrt{10}$

B $\frac{7}{2}$

C $\sqrt{14}$

D $\sqrt{15}$

E 4

11. Suppose that (u_n) is a sequence of real numbers satisfying $u_{n+2} = 2u_{n+1} + u_n$, and that $u_3 = 9$ and $u_6 = 128$. What is u_5?

A 40

B 53

C 68

D 88

E 104

12. Postman Pete has a pedometer to count his steps. The pedometer records up to 99999 steps, then flips over to 00000 on the next step. Pete plans to determine his mileage for a year. On January 1 Pete sets the pedometer to 00000. During the year, the pedometer flips from 99999 to 00000 forty-four times. On December 31 the pedometer reads 50000. Pete takes 1800 steps per mile. Which of the following is closest to the number of miles Pete walked during the year?

A 2500

B 3000

C 3500

D 4000

E 4500

13. For each positive integer n, the mean of the first n terms of a sequence is n. What is the 2008th term of the sequence?

A 2008

B 4015

C 4016

D $4{,}030{,}056$

E $4{,}032{,}064$

14. Triangle OAB has $O = (0,0)$, $B = (5,0)$, and A in the first quadrant. In addition, $\angle ABO = 90^\circ$ and $\angle AOB = 30^\circ$. Suppose that $\overline{OA}$ is rotated 90° counterclockwise about O. What are the coordinates of the image of A?

A $\left(-\frac{10}{3}\sqrt{3},\ 5\right)$

B $\left(-\frac{5}{3}\sqrt{3},\ 5\right)$

C $\left(\sqrt{3},\ 5\right)$

D $\left(\frac{5}{3}\sqrt{3},\ 5\right)$

E $\left(\frac{10}{3}\sqrt{3},\ 5\right)$

15. How many right triangles have integer leg lengths a and b and a hypotenuse of length $b+1$, where $b<100$?

A 6

B 7

C 8

D 9

E 10

16. Two fair coins are to be tossed once. For each head that results, one fair die is to be rolled. What is the probability that the sum of the die rolls is odd? (Note that if no die is rolled, the sum is 0.)

A $\frac{3}{8}$

B $\frac{1}{2}$

C $\frac{43}{72}$

D $\frac{5}{8}$

E $\frac{2}{3}$

17. A poll shows that 70% of all voters approve of the mayor's work. On three separate occasions a pollster selects a voter at random. What is the probability that on exactly one of these three occasions the voter approves of the mayor's work?

A 0.063

B 0.189

C 0.233

D 0.333

E 0.441

18. Bricklayer Brenda would take 9 hours to build a chimney alone, and bricklayer Brandon would take 10 hours to build it alone. When they work together, they talk a lot, and their combined output is decreased by 10 bricks per hour. Working together, they build the chimney in 5 hours. How many bricks are in the chimney?

A 500

B 900

C 950

D 1000

E 1900

19. A cylindrical tank with radius 4 feet and height 9 feet is lying on its side. The tank is filled with water to a depth of 2 feet. What is the volume of the water, in cubic feet?

A $24\pi - 36\sqrt{2}$

B $24\pi - 24\sqrt{3}$

C $36\pi - 36\sqrt{3}$

D $36\pi - 24\sqrt{2}$

E $48\pi - 36\sqrt{3}$

20. The faces of a cubical die are marked with the numbers $1, 2, 2, 3, 3,$ and $4.$ The faces of a second cubical die are marked with the numbers $1, 3, 4, 5, 6,$ and $8.$ Both dice are thrown. What is the probability that the sum of the two top numbers will be $5, 7,$ or $9?$

A $\frac{5}{18}$

B $\frac{7}{18}$

C $\frac{11}{18}$

D $\frac{3}{4}$

E $\frac{8}{9}$

21. Ten chairs are evenly spaced around a round table and numbered clockwise from 1 through 10. Five married couples are to sit in the chairs with men and women alternating, and no one is to sit either next to or directly across from his or her spouse. How many seating arrangements are possible?

A 240

B 360

C 480

D 540

E 720

22. Three red beads, two white beads, and one blue bead are placed in a line in random order. What is the probability that no two neighboring beads are the same color?

A $\frac{1}{12}$

B $\frac{1}{10}$

C $\frac{1}{6}$

D $\frac{1}{3}$

E $\frac{1}{2}$

23. A rectangular floor measures a feet by b feet, where a and b are positive integers with $b > a$. An artist paints a rectangle on the floor with the sides of the rectangle parallel to the sides of the floor. The unpainted part of the floor forms a border of width 1 foot around the painted rectangle and occupies half the area of the entire floor. How many possibilities are there for the ordered pair (a, b)?

A 1

B 2

C 3

D 4

E 5

24. Quadrilateral $ABCD$ has $AB = BC = CD, \angle ABC = 70^\circ$, and $\angle BCD = 170^\circ$. What is the degree measure of $\angle BAD$?

A 75

B 80

C 85

D 90

E 95

25. Michael walks at the rate of 5 feet per second on a long straight path. Trash pails are located every 200 feet along the path. A garbage truck travels at 10 feet per second in the same direction as Michael and stops for 30 seconds at each pail. As Michael passes a pail, he notices the truck ahead of him just leaving the next pail. How many times will Michael and the truck meet?

A 4

B 5

C 6

D 7

E 8

Solutions: https://live.poshenloh.com/past-contests/amc10/2008B/solutions