2010 AMC 10B

Time limit: 75 minutes

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1. What is the value of the following expression?

$$100(100-3)-(100\cdot 100-3)$$

A $-20{,}000$

B $-10{,}000$

C -297

D -6

E 0

2. Makayla attended two meetings during her 9-hour work day. The first meeting took 45 minutes and the second meeting took twice as long. What percent of her work day was spent attending meetings?

A 15

B 20

C 25

D 30

E 35

3. A drawer contains red, green, blue, and white socks with at least 2 of each color. What is the minimum number of socks that must be pulled from the drawer to guarantee a matching pair?

A 3

B 4

C 5

D 8

E 9

4. For a real number x, define $\heartsuit(x)$ to be the average of x and x^2. What is the value of the following expression?

$$\heartsuit(1) + \heartsuit(2) + \heartsuit(3)$$

A 3

B 6

C 10

D 12

E 20

5. A month with 31 days has the same number of Mondays and Wednesdays. How many of the seven days of the week could be the first day of this month?

A 2

B 3

C 4

D 5

E 6

6. A circle is centered at O, $\overline{AB}$ is a diameter and C is a point on the circle with $\angle COB = 50^\circ$. What is the degree measure of $\angle CAB$?

A 20

B 25

C 45

D 50

E 65

7. A triangle has side lengths $10, 10,$ and $12.$ A rectangle has width 4 and area equal to the area of the triangle. What is the perimeter of this rectangle?

A 16

B 24

C 28

D 32

E 36

8. A ticket to a school play costs x dollars, where x is a whole number. A group of 9th graders buys tickets costing a total of $\$48,$ and a group of 10th graders buys tickets costing a total of $\$64.$ How many values for x are possible?

A 1

B 2

C 3

D 4

E 5

9. Lucky Larry's teacher asked him to substitute numbers for $a, b, c, d,$ and e in the expression

$$a-(b-(c-(d+e)))$$

and evaluate the result. Larry ignored the parentheses but added and subtracted correctly and obtained the correct result by coincidence. The numbers Larry substituted for $a, b, c,$ and d were $1, 2, 3,$ and $4,$ respectively. What number did Larry substitute for e?

A -5

B -3

C 0

D 3

E 5

10. Shelby drives her scooter at a speed of 30 miles per hour if it is not raining, and 20 miles per hour if it is raining. Today she drove in the sun in the morning and in the rain in the evening, for a total of 16 miles in 40 minutes. How many minutes did she drive in the rain?

A 18

B 21

C 24

D 27

E 30

11. A shopper plans to purchase an item that has a listed price greater than $\$100$ and can use any one of the three coupons. Coupon A gives 15% off the listed price, Coupon B gives $\$30$ off the listed price, and Coupon C gives 25% off the amount by which the listed price exceeds $\$100$.

Let x and y be the smallest and largest prices, respectively, for which Coupon A saves at least as many dollars as Coupon B or C. What is $y - x$?

A 50

B 60

C 75

D 80

E 100

12. At the beginning of the school year, 50% of all students in Mr. Wells' math class answered "Yes" to the question "Do you love math", and 50% answered "No." At the end of the school year, 70% answered "Yes" and 30% answered "No." Altogether, $x\%$ of the students gave a different answer at the beginning and end of the school year. What is the difference between the maximum and the minimum possible values of x?

A 0

B 20

C 40

D 60

E 80

13. What is the sum of all the solutions of the equation below?

$$x = |2x - |60 - 2x||$$

A 32

B 60

C 92

D 120

E 124

14. The average of the numbers $1, 2, 3, \cdots, 98, 99$, and x is $100x$. What is x?

A $\frac{49}{101}$

B $\frac{50}{101}$

C $\frac{1}{2}$

D $\frac{51}{101}$

E $\frac{50}{99}$

15. On a 50-question multiple choice math contest, students receive 4 points for a correct answer, 0 points for an answer left blank, and -1 point for an incorrect answer. Jesse's total score on the contest was 99. What is the maximum number of questions that Jesse could have answered correctly?

A 25

B 27

C 29

D 31

E 33

16. A square of side length 1 and a circle of radius $\frac{\sqrt{3}}{3}$ share the same center. What is the area inside the circle, but outside the square?

A $\frac{\pi}{3} - 1$

B $\frac{2\pi}{9} - \frac{\sqrt{3}}{3}$

C $\frac{\pi}{18}$

D $\frac{1}{4}$

E $\frac{2\pi}{9}$

17. Every high school in the city of Euclid sent a team of 3 students to a math contest. Each participant in the contest received a different score. Andrea's score was the median among all students, and hers was the highest score on her team. Andrea's teammates Beth and Carla placed 37th and 64th, respectively. How many schools are in the city?

A 22

B 23

C 24

D 25

E 26

18. Positive integers a, b, and c are randomly and independently selected with replacement from the set $\{1, 2, 3, \ldots, 2010\}$.

What is the probability that $abc + ab + a$ is divisible by 3?

A $\frac{1}{3}$

B $\frac{29}{81}$

C $\frac{31}{81}$

D $\frac{11}{27}$

E $\frac{13}{27}$

19. A circle with center O has area 156π. Triangle ABC is equilateral, $\overline{BC}$ is a chord on the circle, $OA = 4\sqrt{3}$, and point O is outside $\triangle ABC$. What is the side length of $\triangle ABC$?

A $2\sqrt{3}$

B 6

C $4\sqrt{3}$

D 12

E 18

20. Two circles lie outside regular hexagon $ABCDEF$. The first is tangent to $\overline{AB}$, and the second is tangent to $\overline{DE}$. Both are tangent to lines BC and FA. What is the ratio of the area of the second circle to that of the first circle?

A 18

B 27

C 36

D 81

E 108

21. A palindrome between 1000 and $10{,}000$ is chosen at random. What is the probability that it is divisible by 7?

A $\frac{1}{10}$

B $\frac{1}{9}$

C $\frac{1}{7}$

D $\frac{1}{6}$

E $\frac{1}{5}$

22. Seven distinct pieces of candy are to be distributed among three bags. The red bag and the blue bag must each receive at least one piece of candy; the white bag may remain empty. How many arrangements are possible?

A 1930

B 1931

C 1932

D 1933

E 1934

23. The entries in a 3×3 array include all the digits from 1 through 9, arranged so that the entries in every row and column are in increasing order. How many such arrays are there?

A 18

B 24

C 36

D 42

E 60

24. A high school basketball game between the Raiders and Wildcats was tied at the end of the first quarter. The number of points scored by the Raiders in each of the four quarters formed an increasing geometric sequence, and the number of points scored by the Wildcats in each of the four quarters formed an increasing arithmetic sequence. At the end of the fourth quarter, the Raiders had won by one point. Neither team scored more than 100 points. What was the total number of points scored by the two teams in the first half?

A 30

B 31

C 32

D 33

E 34

25. Let $a > 0$, and let $P(x)$ be a polynomial with integer coefficients such that

$$\begin{aligned} P(1) &= P(3) \\ &= P(5) = P(7) = a, \end{aligned}$$

and

$$\begin{aligned} P(2) = P(4) &= P(6) \\ &= P(8) = -a. \end{aligned}$$

What is the smallest possible value of a?

A 105

B 315

C 945

D $7!$

E $8!$

Solutions: https://live.poshenloh.com/past-contests/amc10/2010B/solutions