2010 AMC 8

Time limit: 40 minutes

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https://live.poshenloh.com/past-contests/amc8/2010

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1. At Euclid Middle School the mathematics teachers are Miss Germain, Mr. Newton, and Mrs. Young. There are 11 students in Miss Germain's class, 8 students in Mr. Newton's class, and 9 students in Mrs. Young's class taking the AMC 8 Contest this year. How many mathematics students at Euclid Middle School are taking the contest?

A 26

B 27

C 28

D 29

E 30

2. If $a * b = \dfrac{a \times b}{a + b}$ for a, b positive integers, then what is $5 * 10$?

A $\dfrac{3}{10}$

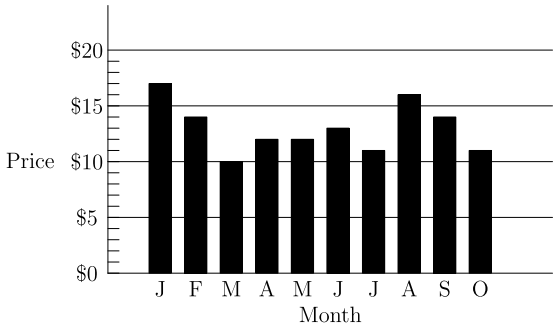
B 1

C 2

D $\dfrac{10}{3}$

E 50

3. The graph shows the price of five gallons of gasoline during the first ten months of the year. By what percent is the highest price more than the lowest price?



A 50

B 62

C 70

D 89

E 100

4. What is the sum of the mean, median, and mode of the numbers $2, 3, 0, 3, 1, 4, 0, 3$?

A 6.5

B 7

C 7.5

D 8.5

E 9

5. Alice needs to replace a light bulb located 10 centimeters below the ceiling in her kitchen. The ceiling is 2.4 meters above the floor. Alice is 1.5 meters tall and can reach 46 centimeters above the top of her head. Standing on a stool, she can just reach the light bulb. What is the height of the stool, in centimeters?

A 32

B 34

C 36

D 38

E 40

6. Which of the following figures has the greatest number of lines of symmetry?

A equilateral triangle

B non-square rhombus

C non-square rectangle

D isosceles trapezoid

E square

7. Using only pennies, nickels, dimes, and quarters, what is the smallest number of coins Freddie would need so he could pay any amount of money less than a dollar?

A 6

B 10

C 15

D 25

E 99

8. As Emily is riding her bicycle on a long straight road, she spots Emerson skating in the same direction $\frac{1}{2}$ mile in front of her. After she passes him, she can see him in her rear view mirror until he is $\frac{1}{2}$ mile behind her. Emily rides at a constant rate of 12 miles per hour, and Emerson skates at a constant rate of 8 miles per hour. For how many minutes can Emily see Emerson?

A 6

B 8

C 12

D 15

E 16

9. Ryan got 80% of the problems correct on a 25-problem test, 90% on a 40-problem test, and 70% on a 10-problem test. What percent of all the problems did Ryan answer correctly?

A 63

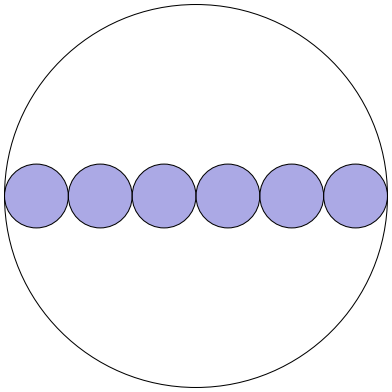
B 75

C 80

D 84

E 86

10. Six pepperoni circles will exactly fit across the diameter of a 12-inch pizza when placed as shown. If a total of 24 circles of pepperoni are placed on this pizza without overlap, what fraction of the pizza is covered by pepperoni?



A $\frac{1}{2}$

B $\frac{2}{3}$

C $\frac{3}{4}$

D $\frac{5}{6}$

E $\frac{7}{8}$

11. The top of one tree is 16 feet higher than the top of another tree. The heights of the two trees are in the ratio $3:4$. In feet, how tall is the taller tree?

A 48

B 64

C 80

D 96

E 112

12. Of the 500 balls in a large bag, 80% are red and the rest are blue. How many of the red balls must be removed so that 75% of the remaining balls are red?

A 25

B 50

C 75

D 100

E 150

13. The lengths of the sides of a triangle in inches are three consecutive integers. The length of the shortest side is 30% of the perimeter. What is the length of the longest side?

A 7

B 8

C 9

D 10

E 11

14. What is the sum of the prime factors of 2010?

A 67

B 75

C 77

D 201

E 210

15. A jar contains 5 different colors of gum drops. 30% are blue, 20% are brown, 15% are red, 10% are yellow, and the other 30 gum drops are green. If half of the blue gum drops are replaced by brown gum drops, how many gum drops will be brown?

A 35

B 36

C 42

D 48

E 64

16. A square and a circle have the same area. What is the ratio of the side length of the square to the radius of the circle?

A $\frac{\sqrt{\pi}}{2}$

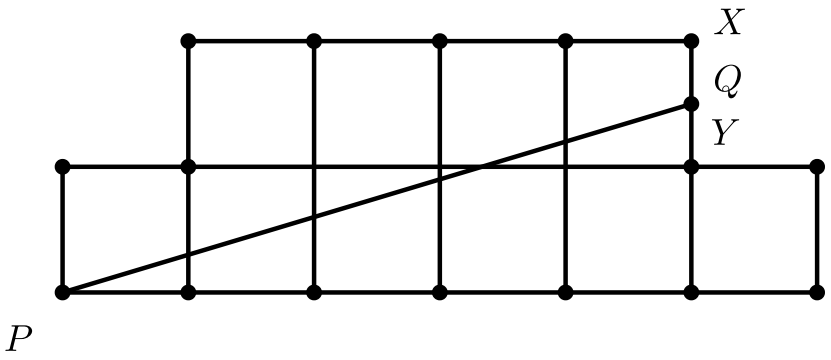
B $\sqrt{\pi}$

C π

D 2π

E π^2

17. The diagram shows an octagon consisting of 10 unit squares. The portion below $\overline{PQ}$ is a unit square and a triangle with base 5. If $\overline{PQ}$ bisects the area of the octagon, what is the ratio $\dfrac{XQ}{QY}$?



A $\dfrac{2}{5}$

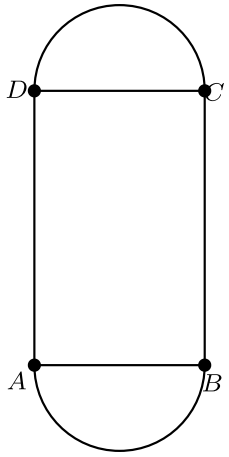
B $\dfrac{1}{2}$

C $\dfrac{3}{5}$

D $\dfrac{2}{3}$

E $\dfrac{3}{4}$

18. A decorative window is made up of a rectangle with semicircles on either end. The ratio of AD to AB is $3:2$, and $AB = 30$ inches. What is the ratio of the area of the rectangle to the combined areas of the semicircles?



A $2:3$

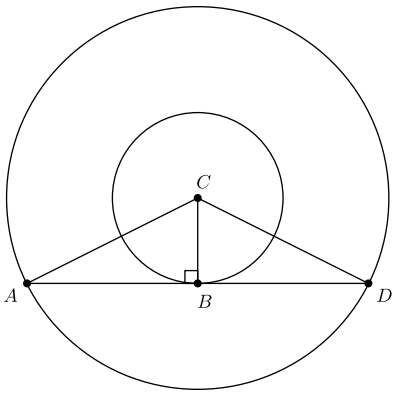
B $3:2$

C $6:\pi$

D $9:\pi$

E $30:\pi$

19. The two circles pictured have the same center C. Chord $\overline{AD}$ is tangent to the inner circle at B, AC is 10, and chord $\overline{AD}$ has length 16. What is the area between the two circles?



A 36π

B 49π

C 64π

D 81π

E 100π

20. In a room, $\frac{2}{5}$ of the people are wearing gloves, and $\frac{3}{4}$ of the people are wearing hats. What is the minimum number of people in the room wearing both a hat and gloves?

A 3

B 5

C 8

D 15

E 20

21. Hui is an avid reader. She bought a copy of the bestseller Math is Beautiful. On the first day, Hui read $\frac{1}{5}$ of the pages plus 12 more, and on the second day she read $\frac{1}{4}$ of the remaining pages plus 15 pages. On the third day she read $\frac{1}{3}$ of the remaining pages plus 18 pages. She then realized that there were only 62 pages left to read, which she read the next day. How many pages are in this book?

A 120

B 180

C 240

D 300

E 360

22. The hundreds digit of a three-digit number is 2 more than the units digit. The digits of the three-digit number are reversed, and the result is subtracted from the original three-digit number. What is the units digit of the result?

A 0

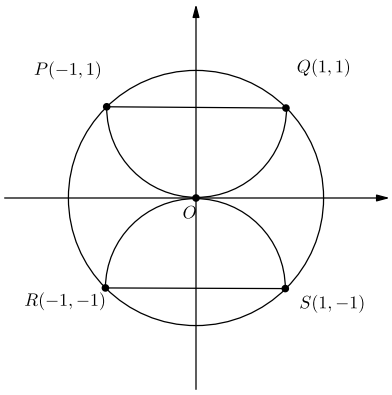
B 2

C 4

D 6

E 8

23. Semicircles POQ and ROS pass through the center O. What is the ratio of the combined areas of the two semicircles to the area of circle O?



A $\frac{\sqrt{2}}{4}$

B $\frac{1}{2}$

C $\frac{2}{\pi}$

D $\frac{2}{3}$

E $\frac{\sqrt{2}}{2}$

24. What is the correct ordering of the three numbers, $10^8, 5^{12}$, and 2^{24}?

A 2^{24} < 10^8 < 5^{12}

B 2^{24} < 5^{12} < 10^8

C 5^{12} < 2^{24} < 10^8

D 10^8 < 5^{12} < 2^{24}

E 10^8 < 2^{24} < 5^{12}

25. Every day at school, Jo climbs a flight of 6 stairs. Jo can take the stairs 1, 2, or 3 at a time. For example, Jo could climb 3, then 1, then 2. In how many ways can Jo climb the stairs?

A 13

B 18

C 20

D 22

E 24

Solutions: https://live.poshenloh.com/past-contests/amc8/2010/solutions